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"spin faster," i.e., increasing the velocity of the circulation, decreasing the time required for a given mass to complete the cycle. In either case he has increased the energy turn-over per unit of time. Whether, in this he has been unconsciously fulfilling one of those laws of nature according to which certain quantities tend toward a maximum, is a question well deserving of our attention.

* Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 44.

¹Lotka, A. J., *Physic. Rev.*, **24**, 1912 (235-238); *J. Washington Acad. Sci.*, **2**, 1912 (2, 49, 66); *Science Progress*, **55**, 1920 (406-417); *Proc. Am. Acad. Arts Sci.*, **55**, 1920 (237-153); these PROCEEDINGS, *Sci.* **6**, 1910 (410-415).

²*Proc. Amer. Acad.*, loc cit., p. 142.

³See for example Picard, *Traité d'Analyse*; H. Bateman, *Differential Equations*, 1918, p. 245.

⁴Spencer, *First Principles*, Chapter XXII; Winiarskie, "Essai sur la Mécanique Sociale," *Revue Philosophique*, **44**, 1900 (113).

⁵At the time of reading proof this project is partially realized. A discussion of the applicability of the Le Chatelier principle to systems of the general character here considered will appear in a forthcoming issue of the *Proceedings of the American Academy of Arts and Sciences*

⁶Lotka, A. J., *London, Phil. Mag.*, Aug., 1911, p. 353.

A FORMULA FOR THE VISCOSITY OF LIQUIDS¹

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1. In this paper I obtain for the viscosity of a liquid the formula

$$\eta = \frac{n N h}{2M(v-\delta)}, \quad (1)$$

where N is the number of molecules in a mol, h is Planck's constant, M is the molecular weight of the liquid in the gas phase, v its volume per gram, and n an integer. The quantity δ is the co-volume as used in the equation of state of Keyes¹

$$p = \frac{RT}{v-\delta} - \frac{A}{(v-l)^2}, \quad (2)$$

In all the cases to which I have applied the formula, $n=6$ and so (1) takes the form

$$\eta(v-\delta) = 3Nh/M \quad (3)$$

It is to be noted that $3N/M$ is the number of translational degrees of freedom of the molecules in the volume v of the liquid.

2. To prove equation (1), let x, y, z be rectangular coördinates and suppose the liquid to flow parallel to the x -axis in such a way that, u_0 being

the velocity at the point (x, y, z) ,

$$\partial u_0 / \partial z = 1. \quad (4)$$

Let the components of velocity of a molecule parallel to the x and z axes be u, w respectively. When a molecule of mass m moves in the positive direction of the z -axis across the xy -plane, the x -component of momentum transferred across that plane is mu . When it crosses in the negative direction the transfer is $-mu$. Under these conditions the viscosity η of the liquid is defined as the total x -component of momentum transferred across unit area of the xy -plane in unit time.² To find the viscosity we therefore find the number of molecules of each velocity crossing unit area and multiply by the average momentum transferred by each.

Let $f(w) dw$ be the number of molecules per cubic centimeter with components of velocity parallel to the z -axis between w and $w + dw$. If the molecules did not influence each other, the number of these that would cross unit area of the xy -plane in unit time would be

$$w f(w) dw. \quad (5)$$

Because of the interference of molecules with each other, this must be replaced by

$$\frac{v}{v-\delta} w f(w) dw. \quad (6)$$

To see this we note that if the molecules moved independently, the z -component of momentum transferred across unit area of the xy -plane in both directions in unit time would be the pressure

$$RT/v. \quad (7)$$

From equation (2), in the real liquid this is replaced by

$$RT/(v-\delta). \quad (8)$$

Now the mass of the molecule and the temperature (average kinetic energy) are the same in both cases. Hence the increased momentum (8) compared with (7), can only be due to an increase in the number of molecules crossing in unit time. Since the same law of distribution of velocities (Maxwell's Law) is assumed in both cases, the ratio in which the numbers are increased must be the same $v/(v-\delta)$ for all velocities. Thus the number of molecules with velocity components between w and $w + dw$ crossing unit area of the xy -plane in unit time is given by equation (6).

The velocity u of a given molecule parallel to the x -axis will not always be equal to the average velocity u_0 of the liquid at that point. There will, however, be a series of instants (which we may call collisions) when u will be equal to u_0 . I assume that the interval between two such instants will be a half period (interval between libration limits)³ in the sense of the quantum theory, and that the molecules can be treated as moving with constant velocity between consecutive collisions. Suppose then a molecule reaching the xy -plane has been moving a time, t , since its last collision.

From (4) its momentum parallel to the x -axis due to the motion of the liquid at that point will be $m w t$.

From (6) the total x -component of momentum transferred in the positive direction across unit area of the xy -plane in unit time will then be

$$\frac{v}{v-\delta} \int_0^\infty m w^2 t f(w) dw.$$

There will be an equal transfer due to molecules moving in the negative direction. Hence

$$\eta = \frac{v}{v-\delta} \int_0^\infty 2m w^2 t f(w) dw. \quad (9)$$

The time t can be considered as the interval between collisions at times⁴ t_0 , t_1 . Since the molecule is assumed to move with constant velocity between collisions,

$$2m w^2 t = 2 \int_{t_0}^{t_1} m w^2 dt = nh, \quad (10)$$

where, according to Sommerfeld's theory, n is an integer. Also

$$v \int_0^\infty f(w) dw = N/2M, \quad (11)$$

since it is the number of molecules per gram for which w is positive. Combining (9), (10), and (11), we get (1) which was to be proved.

3. Owing to lack of data on the equation of state, the only substances on which the formula can at present be tested are carbon dioxide, ether, and mercury. Using the values of N and h given by Birge,⁵

$$3Nh = 3(6.0594)(6.5543)10^{-4} = .011914,$$

and so equation (3) takes the form

$$\eta(v-\delta) = .011914/M. \quad (12)$$

Values of δ for carbon dioxide and ether,⁶ tables I and II, were supplied by Professor Keyes. At low temperatures the measured viscosities and those calculated by equation (12) do not differ by more than the experimental error. In case of carbon dioxide the temperature 30° is too near the critical point ($t=31^\circ$) for satisfactory use of the equation of state. Above 10° the calculated viscosity of ether is too large, the difference increasing with the temperature. This may be due to the fact that ether is a complex of more than one type of molecule. The equation of state was determined on the assumption that each liquid molecule is formed by the combination of two gas molecules. This may be substantially true at 10° and not at 100° .

To obtain values of $v-\delta$ for mercury, I make use of the fact that monatomic substances seem to have constant co-volumes⁷ δ . Since mercury is monatomic in the gas phase, I assume that δ is constant or nearly constant in the liquid phase. Also the term

$$RT/(v-\delta)$$

in case of mercury is very large (more than 15000 atmospheres). Hence at atmospheric pressures we may neglect p and so write (2) in the form

$$\sqrt{\frac{V-\delta}{T}} = c (v-l)$$

where c is constant. Differentiating this equation with respect to v and then with respect to T , assuming δ constant, we get

$$(v-\delta)^2 \left(3 \frac{dv}{dt} + 2T \frac{d^2v}{dt^2} \right) - 2T(v-\delta) \left(\frac{dv}{dt} \right)^2 - T^2 \left(\frac{dv}{dt} \right)^3 = 0,$$

whence

$$V-\delta = T \frac{dv}{dt} \left[\frac{\frac{dv}{dt} + \sqrt{4 \left(\frac{dv}{dt} \right)^2 + 2T \frac{d^2v}{dt^2}}}{3 \frac{dv}{dt} + 2T \frac{d^2v}{dt^2}} \right]. \quad (13)$$

According to Callendar and Moss⁸

$$\frac{V}{V_0} = 1 + 1805553 \left(\frac{t}{100} \right) 10^{-8} + 12444 \left(\frac{t}{100} \right)^2 10^{-8} + 2539 \left(\frac{t}{100} \right)^3 10^{-8} \quad (14)$$

is the ratio of the volume of mercury at t° to its volume at 0° . The values of $v-\delta$ in table III are obtained by calculating the derivatives of v from (14) and substituting in (13). Below 100° the difference of the calculated viscosities and those measured is not greater than the difference between the values obtained by different observers. At higher temperatures the calculated viscosities are consistently smaller than those measured. The observations, however, differ greatly, owing probably to oxidation of the mercury in contact with the air. The result may be to increase the viscosity by almost any amount. This work should be repeated with mercury of the greatest purity in a vacuum or in contact with some inactive gas.

TABLE I
CARBON DIOXIDE

$$\log \delta = .09780 - \frac{0.1978}{v}, \quad \eta(v-\delta) = .0002708$$

TEMP.	PRESSURE	v	$v-\delta$	$\eta \times 10^6$ CAL.	$\eta \times 10^6$ OBS.	OBSERVER
5	Sat.	1.113	.278	974	925	Warburg and Babo ⁹
10	"	1.168	.317	854	852	
15	"	1.228	.360	751	784	
20	"	1.306	.419	646	712	
20	59 at.	1.291	.408	664	697	Phillips ¹⁰
20	72	1.224	.358	756	771	
20	83	1.189	.332	816	823	
30	72	1.566	.627	432	458	
30	90	1.339	.445	608	643	
30	104	1.274	.395	686	733	

TABLE II

ETHER

$$\log \delta = .48959 - \frac{.50981}{v}, \quad \eta(v-\delta) = .0001608$$

TEMP.	v	$v-\delta$	$\eta \times 10^8$ CAL.	$\eta \times 10^8$ OBS.	OBSERVER
0°	1.3583	.0573	2806	2860	Thorpe and Roger ¹¹
10	1.3797	.0612	2626	2585	
20	1.4016	.0663	2425	2345	
30	1.4247	.0703	2287	2120	
30	1.4247	.0703	2287	2134	Pound ¹²
30	1.4247	.0703	2287	2150	Heydweiller ¹³
40	1.4506	.0761	2113	1970	
50	1.4784	.0828	1942	1810	
60	1.5019	.0888	1811	1666	
70	1.5309	.0961	1673	1528	
80	1.5620	.1058	1519	1400	
90	1.6000	.1177	1366	1284	
100	1.6380	.1302	1235	1177	

TABLE III

MERCURY

$$\eta(v-\delta) = .00005935$$

TEMP.	$v-\delta$	η -CAL.	η -OBS.	$\eta(v-\delta)$	OBSERVER
-18.1	.0033282	.01784	.01836	.00006111	Koch ¹⁴
0	.0035606	.01667	.01661	.00005914	Plüss ¹⁵
0	.0035606	.01667	.01688	.00006010	Koch
10	.0036885	.01609	.01577	.00005817	Umani
10.1	.0036897	.01609	.01620	.00005975	Koch
14.95	.0037538	.01583	.01571	.00005898	Benard ¹⁶
15.1	.0037557	.01581	.01575	.00005914	
15.7	.0037635	.01578	.01570	.00005909	"
15.9	.0037661	.01577	.01583	.00005961	"
15.95	.0037668	.01577	.01581	.00005954	"
16.1	.0037687	.01576	.01563	.00005889	"
16.2	.0037700	.01575	.01563	.00005892	"
16.5	.0037717	.01574	.01572	.00005929	"
18.76	.0038006	.01562	.01558	.00005921	Plüss
20	.0038165	.01555	.01547	.00005904	"
20	.0038165	.01555	.01579	.00006026	Schweidler
34.05	.0039967	.01485	.01476	.00005899	Plüss
40	.0040732	.01457	.01483	.00006041	Schweidler
62.9	.0043682	.01359	.01360	.00005941	Plüss
79.4	.0045817	.01295	.01299	.00005952	"
97.95	.0048225	.01230	.01263	.00006091	"
99	.0048362	.01227	.01227	.00005934	Koch
124	.0051589	.01151	.01171	.00006041	"
137.5	.0053304	.01114	.01167	.00006221	Plüss
154	.0055456	.01070	.01092	.00006056	Koch
			av.	.00005968	

¹F. G. Keyes, "A New Equation of Continuity," *Proc. Nat. Acad. Sci.*, **3**, 1917 (323-330).

- ² Jeans, *Dynamical Theory of Gases*, p. 247.
³ Adams, "The Quantum Theory," *Bull. Nat. Res. Council*, **1**, No. 5, Oct., 1920.
⁴ Jeans, *loc. cit.*, p. 246.
⁵ Ithaca, *Physic. Rev.*, **14**, p. 368.
⁶ Wm. A. Felsing (Thesis), Technology Press, June, 1918.
⁷ Keyes, *loc. cit.*
⁸ *Proc. R. Soc. London*, **84A**, p. 595.
⁹ Leipzig, *Ann. Physik*, **17A**, 1882 (418).
¹⁰ *Proc. R. Soc. London*, **87A**, pp. 556-57.
¹¹ *Phil. Trans. R. Soc.*, **185A**, p. 572.
¹² *J. Chem. Soc. London*, **99**, p. 708.
¹³ *Ann. Physik*, **59**, p. 200.
¹⁴ Landolt Bornstein (1912).
¹⁵ *Z. Anorg. Chem.*, **93**, p. 18.
¹⁶ Brillouin, *Leçons sur la Viscosité*, pp. 152-159.

THE ANGULAR DIAMETER OF ALPHA BOOTIS BY THE INTERFEROMETER

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Since the measurement of the diameter of Betelgeuse¹ in December, 1920, observations for the determination of stellar diameters have been continued with the 20-foot interferometer attached to the upper end of the 100-inch Hooker reflector.

Several definite results have been obtained, and others, less definite because of poor observing conditions, have been partially checked by comparison with stars whose diameters are known to be too small for measurement with this instrument. Let the visibility of the zero fringes for any particular separation of the mirrors be reduced by seeing which is poor as compared with that on a fine night; it is then hopeless to make final measures. If, however, the interferometer is turned to a neighboring check star and the fringes are seen, and then turned to the star under consideration and they do not appear, the observer is justified in saying that there is a definite decrease in visibility.

The data thus far obtained are as follows: For α Tauri (Aldebaran) fringes of gradually decreasing visibility have been observed at 13, 14.5 and 19 feet. Further observations are necessary between the last two positions to determine whether the fringes vanish between these points or whether a longer beam will be necessary to obtain a measure of the diameter. If the fringes vanish around 16-18 feet, as observations under poor conditions lead one to suspect they will, then the fringes observed at 19 feet must lie beyond the point of disappearance, on the ascending branch of the visibility curve where it rises toward the second maximum.